Numerical Comparison and Utilization of Riemann Gradient Descent

2025 Quantum Machine Learning REU Program

Richard Zheng[1], Kasra Rajabzadeh Dizaji[2], Christian Arenz[2], Glen S. Uehara[3], Andreas Spanias[3]

[1]Department of MATH, Yale University [2]Arenz Lab, Arizona State University [3]SenSIP Center, Arizona State University

BM Quantum Computer

MOTIVATION

- Variational Quantum Algorithms (VQAs) use both quantum and classical computation for optimization
- In VQAs, the quantum part is expensive while the classical part is slow
- Riemann Gradient Descent (RGD) has shown promise in turning the entire optimization algorithm quantum

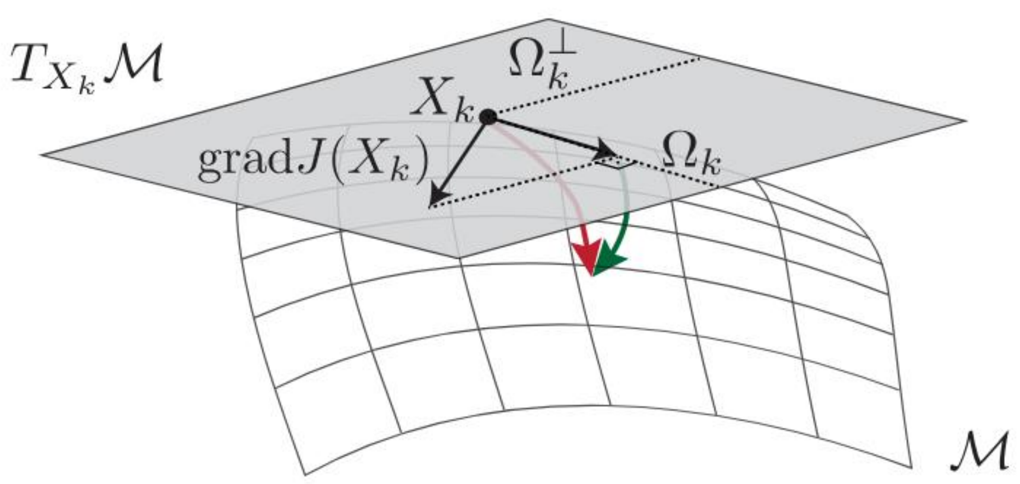

Fig 1: Gradient taken on Riemann Manifold [1]

RESEARCH OBJECTIVES/PLAN

- Develop a numerical simulation of RGD algorithm
  - Test on max cut for different graphs including NP-hard 3 regular graphs
  - Note how RGD scales against Hamiltonian complexity
- Develop quantum circuit implementation of above
- Compare number of steps to convergence between QAOA and RGD

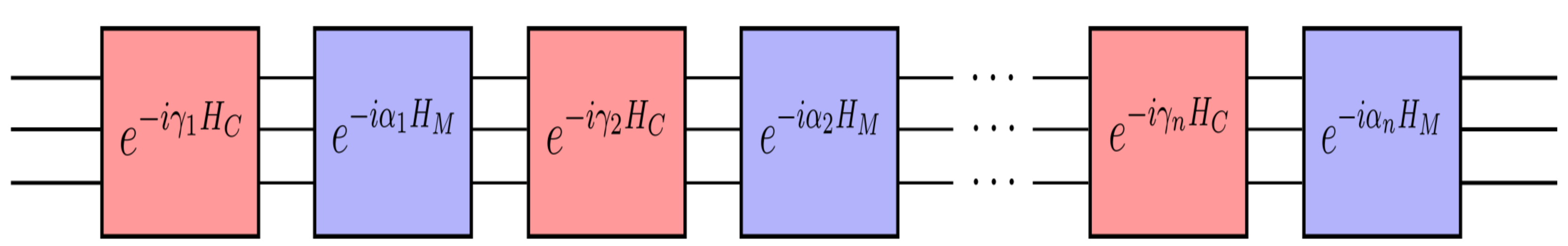

Fig 2: QAOA Circuit from Pennylane.ai

RESULTS AND CONCLUSION

- Ran RGD numerically on:
  - Pauli Operators with different scaling complexities (Fig 4)
  - Max Cut problem with different types of graphs (Fig 5)

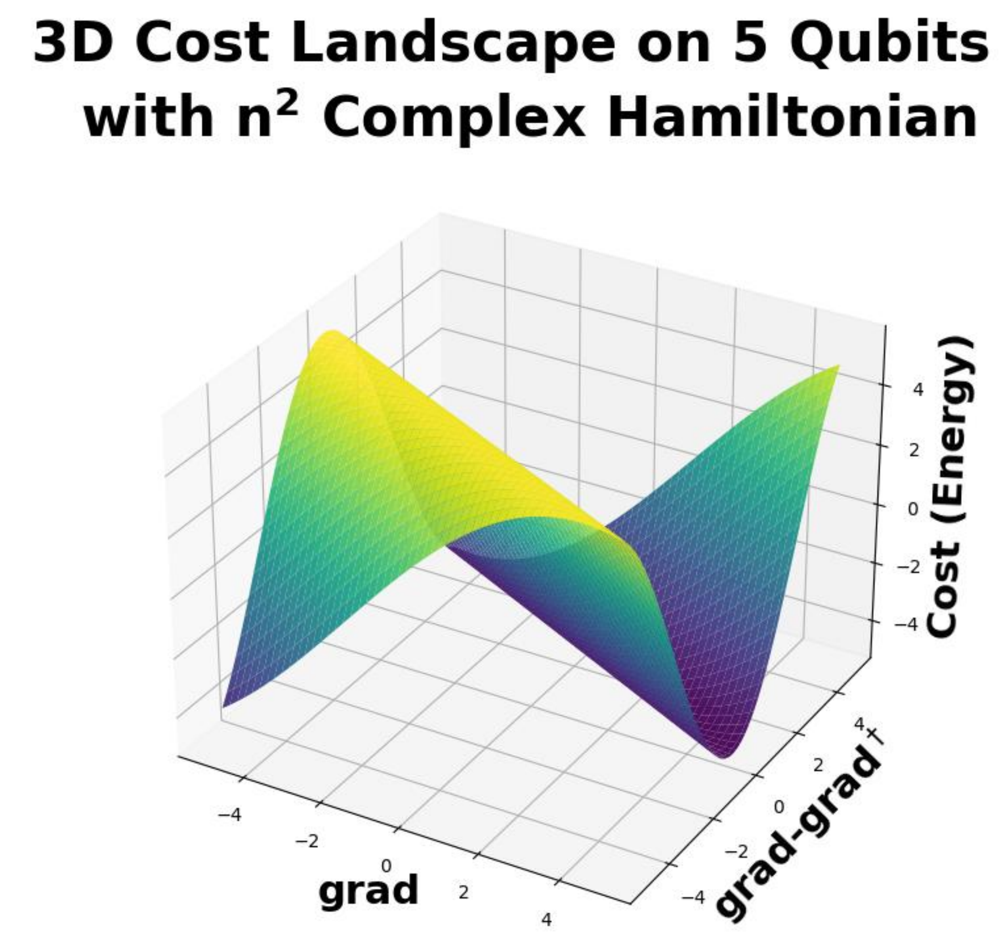

Fig 3: Cost Landscape from the Pauli Complexity Hamiltonian

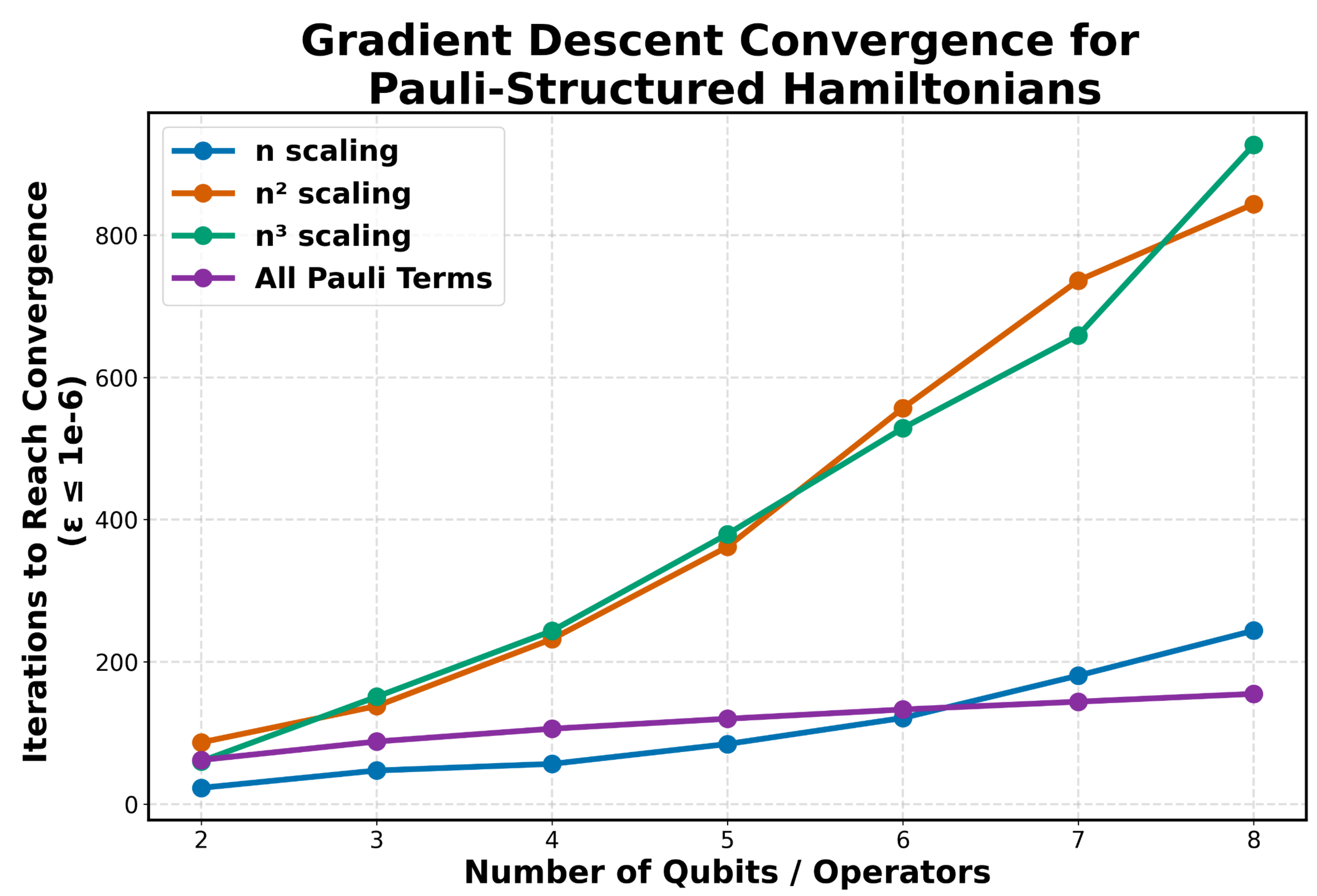

Fig 4: Graph on Hamiltonians Generated by Randomly Populated Paulis

METHODS

- RGD is implemented numerically on Python
- Simulations are run on ASU Sol supercomputer
- RGD uses cost function $\langle\psi_0|U^* H U|\psi_0\rangle$
- RGD calculates gradient as $e^{[|\psi\rangle\langle\psi|, H]}$

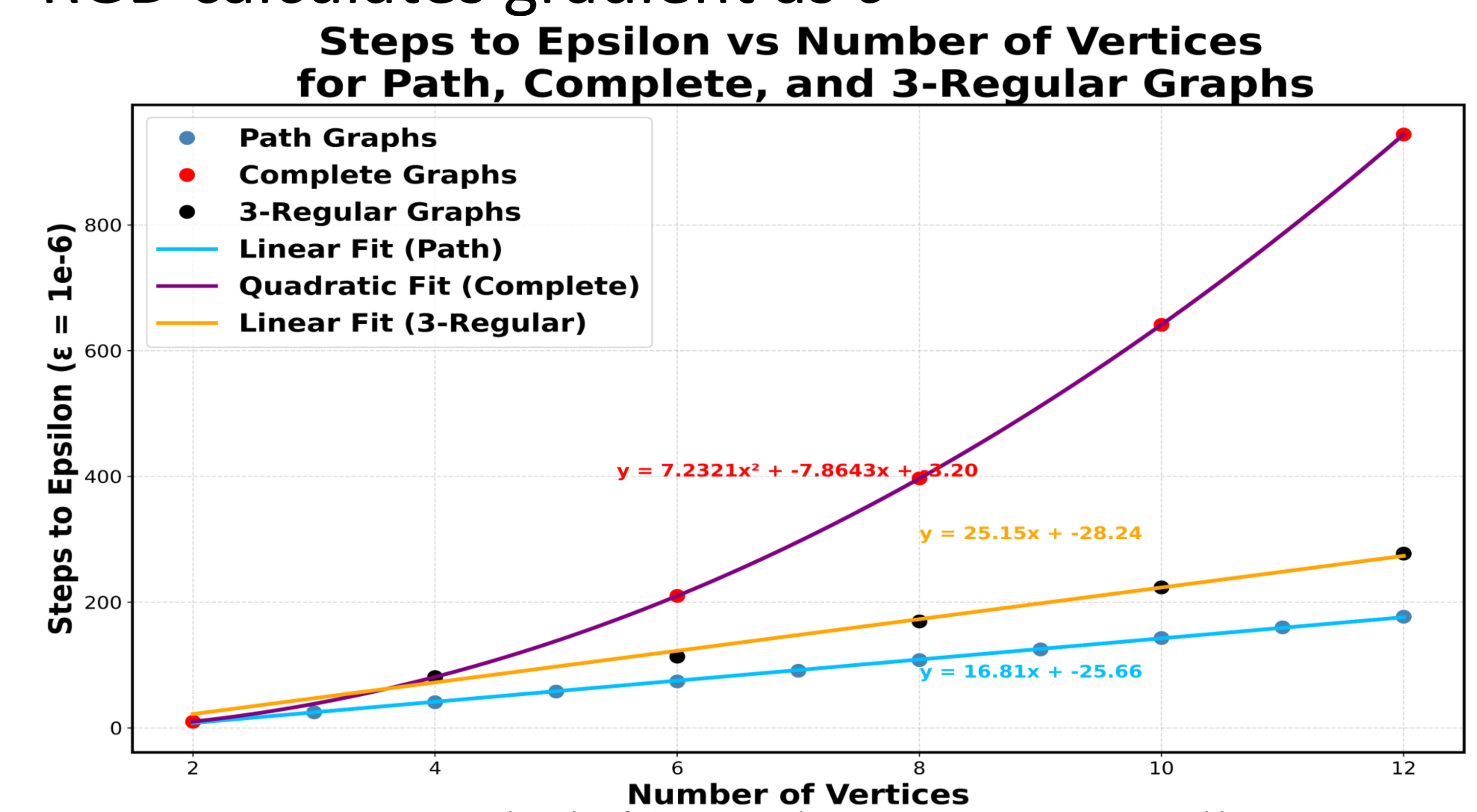

Fig 5: Numerical results of Riemann Gradient Descent on maximum cut problem

NEXT STEPS

- Push the max cut numerical RGD to further vertices
- Analyze trend in scaling Hamiltonian complexity
- Compare parameter shift quantum circuit gradient techniques

CONCLUSIONS

- Found convergence steps scales with spectral norm of Hamiltonians

REFERENCES

[1] N. McMahon, M. Pervez, and C. Arenz, "Equating quantum imaginary time evolution, Riemannian gradient flows, and stochastic implementations", 2025.

[2] M. Cerezo et al., "Variational quantum algorithms," Nature Reviews Physics, 2021.

[3] E. Malvetti, C. Arenz, G. Dirr, and T. Schulte-Herbruggen, "Randomized Gradient Descents on Riemannian Manifolds: Almost Sure Convergence to Global Minima in and beyond Quantum Optimization," 2024.

Ira A. Fulton Schools of Engineering ARIZONA STATE UNIVERSITY

SenSIP Sensor, Signal and Information Processing Center

Sensor Signal and Information Processing Center
https://sensip.asu.edu

Funded in part by NSF Award 2349567