Clarkson

COSINE LAB

Analysis of a modified SEIRS compartmental model for COVID-19

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Arizona State University

SenSIP Industry Consortium

SenSIP Industry-University Meeting
(November 30, 2023)

NSF

Background: Why disease modeling?

- Disease modeling uses mathematical models to predict the spread and severity of diseases
- Captures intervention measures and anticipates the number of people impacted
- Models can be used to conduct epidemiological research, studying patterns of disease transmission and factors influencing its spread
- Compartmental models provide one way to do this

Examples of Compartmental models

Well Known Compartmental models for disease modeling:

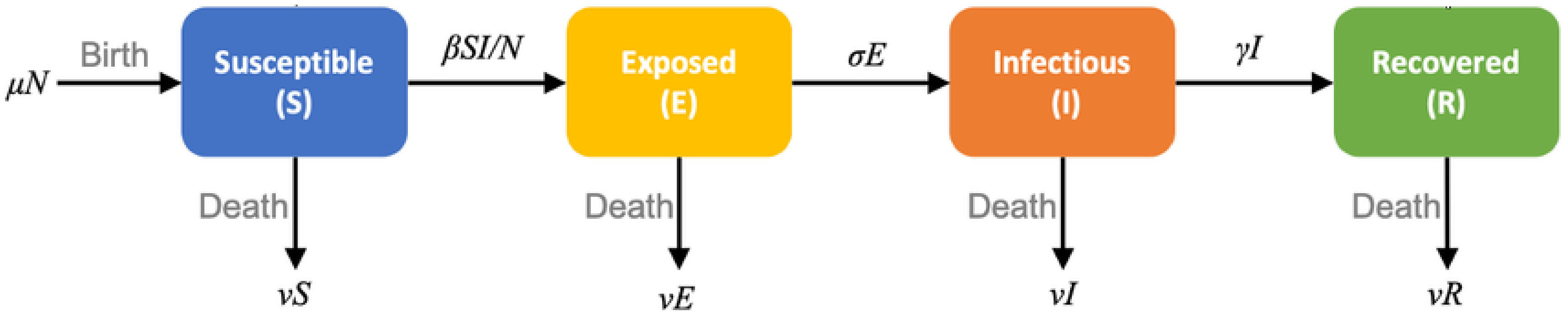
1. SI Model (Susceptible-Infected)
2. SIR Model (Susceptible-Infected- Removed)
3. SEIR Model (Susceptible-Exposed-Infected- Removed)
4. SEIRS Model (Susceptible-Exposed-Infected- Recovered-Susceptible)

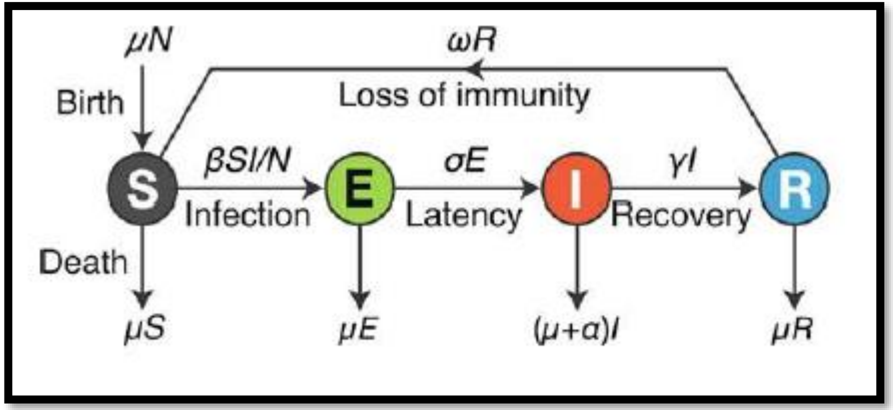
We will focus on the SEIRS model

Definitions: S, E, I, R

1. S - Susceptible: Individuals who are at risk of contracting the disease.
2. E - Exposed: Individuals who have been exposed to the pathogen but are not yet infectious.
3. I - Infected: Individuals who are infected and capable of transmitting the disease to susceptible individuals.
4. R - Recovered: Individuals who have recovered from the disease and are assumed to be immune or no longer able to transmit the disease, Removed – Die from the community.

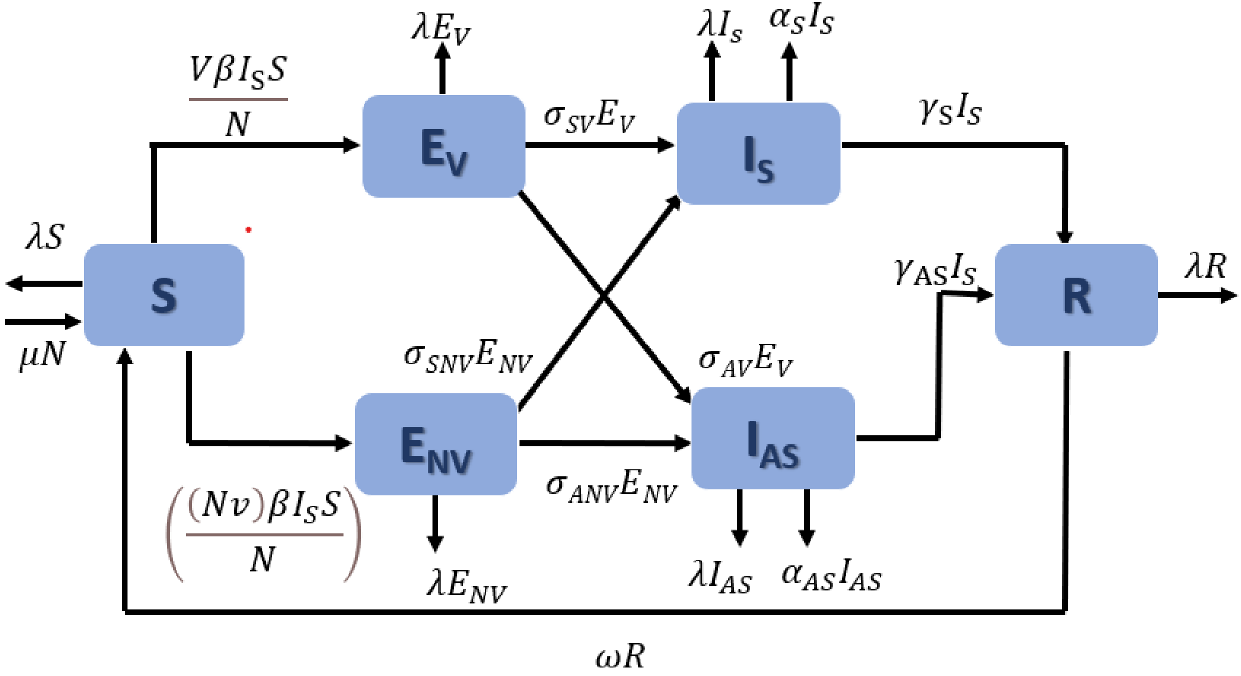
SEIR and SEIRS Model





O. N. Bjørnstad, K. Shea, M. Krzywinski, and N. Altman, “The seirs model for infectious disease dynamics.” Nature methods, vol. 17, no. 6, pp. 557–559, 2020

Flow diagram of a modified SEIRS model



- λ is the natural death rate.
- μ is the natural birth rate.
- β is the contact rate.
- V is the fraction of the population vaccinated against COVID-19.
- NV is the fraction without the vaccine so V+NV = 1.
- α_s , α_{AS} are the death rate due to COVID-19.
- **γ_s , γ_{AS}: the recovery rates affect symptomatic and asymptomatic infected persons, respectively.**
- σ_{SV} , σ_{SNV} , σ_{AV}, and σ_{ANV} are the rates of transition from exposed to infected.
- **ω is the fraction of recovered population who lose immunity and return to susceptible.**

Differential Equation Representations

$$\frac{ds}{dt} = \mu N + \omega R - \lambda S - \left(\frac{V\beta I_S S}{N}\right) - \left(\frac{(NV)\beta I_S S}{N}\right)$$

$$\frac{dI_S}{dt} = \sigma_{SV} E_V + \sigma_{SNV} E_{NV} - (\lambda + \alpha_S + \gamma_S) I_S$$

$$\frac{dE_V}{dt} = \left(\frac{V\beta I_S S}{N}\right) - (\lambda + \sigma_{SV} + \sigma_{AV}) E_V$$

$$\frac{dI_{AS}}{dt} = \sigma_{AV} E_V + \sigma_{ANV} E_{NV} - (\lambda + \alpha_{AS} + \gamma_{AS}) I_{AS}$$

$$\frac{dE_{NV}}{dt} = \left(\frac{(NV)\beta I_S S}{N}\right) - (\lambda + \sigma_{SNV} + \sigma_{ANV}) E_{NV}$$

$$\frac{dR}{dt} = \gamma_S I_S + \gamma_{AS} I_{AS} - (\lambda + \omega) R$$

S. Annas, Muh. Isbar Pratama and Muh. Rifandi et al. / Stability analysis and numerical simulation of SEIR model for pandemic COVID-19 spread in Indonesia, *Chaos, Solitons and Fractals 139 (2020) 110072*

SEIRS Model Time Evolution

Definition: Equilibrium Points

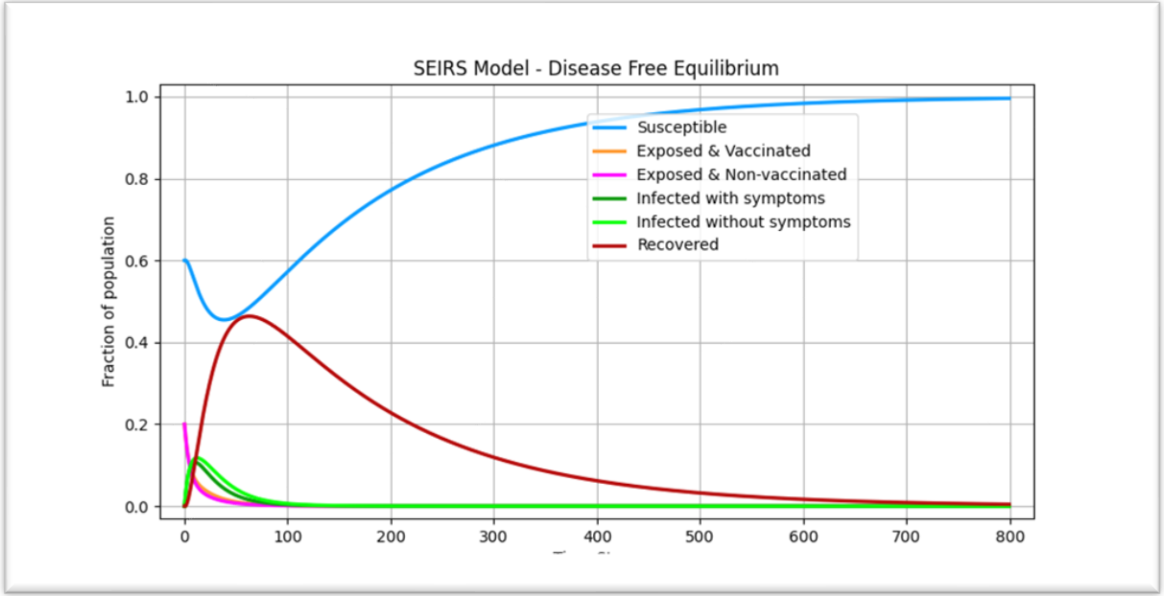
1. An equilibrium point is a state where the system variables (S, E, I, R) remain constant over time.

2. For the modified SEIRS model, we discuss two equilibrium points:

- Disease-free equilibrium (DFE)
- Endemic equilibrium (EE)

3. To solve for equilibrium points, we set the derivatives of the compartment values (with respect to time) to zero.

SEIRS Model : Disease Free Equilibrium



Condition : $I_S = I_{AS} = 0$ (At equilibrium)

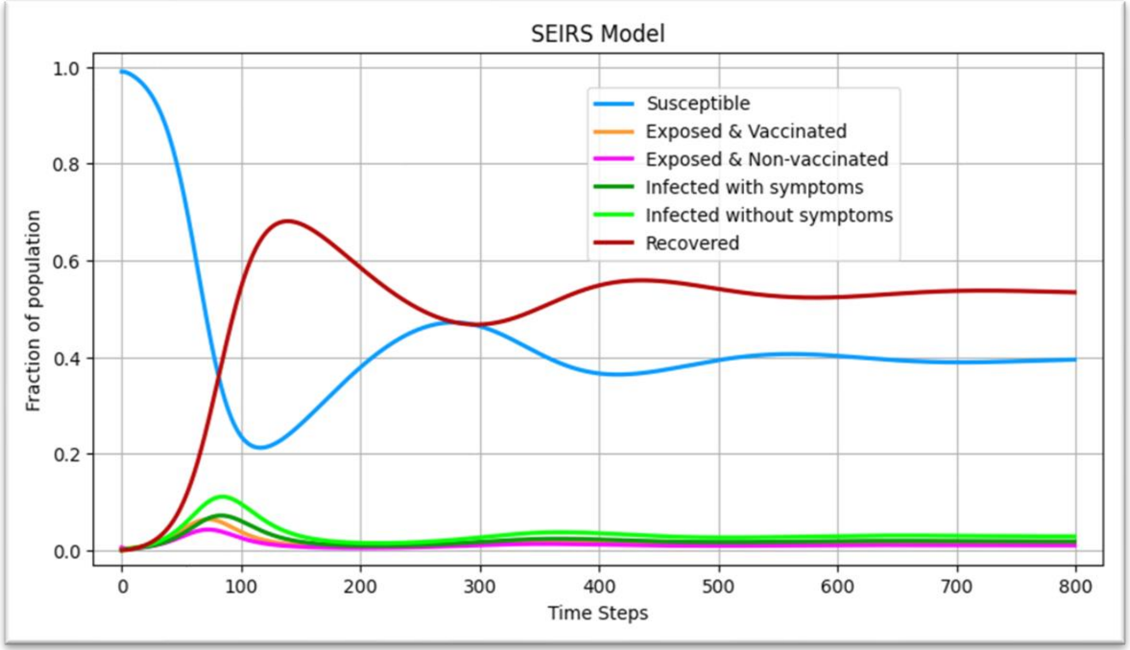
Results: Values of I_S and I_{AS} become zero over time.

Summary:

- ❑ DFE indicates the disease is eradicated from the population.
- ❑ The entire population moves to the suscpetible compartment.

➢ Disease-free Equilibrium converges to (1,0,0,0,0,0)

SEIRS Model : Endemic Equilibrium



Condition : $I_S > 0$, $I_{AS} > 0$ (At equilibrium)

Results: Values of I_S and I_{AS} become non-zero constant over time.

Summary:

- ❑ EE indicates the disease is present among the population.
- ❑ The disease spread remains constant.

➢ Endemic Equilibrium converges to $(S^*, E_V^*, E_{NV}^*, I_S^*, I_{AS}^*, R^*)$

Endemic **E**quilibrium **E**quations

$$S^* = \frac{ABCN}{\sigma_{SV}(V)\beta B + \sigma_{SNV}(NV)\beta A}$$

$$E_V{}^* = \frac{\left(\frac{(V)BC}{\sigma_{SV}(V)B + \sigma_{SNV}(NV)A}\right)\left\{\frac{\lambda ABCN}{\beta} - \mu N(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)\right\}}{\frac{\omega}{(\lambda+\omega)}\left\{\gamma_S(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A) + \frac{\gamma_{AS}C}{D}(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)\right\} - ABC\big((V) + (NV)\big)}$$

$$E_{NV}{}^* = \frac{\left(\frac{(NV)AC}{\sigma_{SV}(V)B + \sigma_{SNV}(NV)A}\right)\left\{\frac{\lambda ABCN}{\beta} - \mu N(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)\right\}}{\frac{\omega}{(\lambda+\omega)}\left\{\gamma_S(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A) + \frac{\gamma_{AS}C}{D}(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)\right\} - ABC\big((V) + (NV)\big)}$$

O. Diekmann, J. A. P Heesterbeek, J.A.J. Metz, On the definition and the computation of the basic reproduction ratio(R0) in the models for the infectious disease in the heterogenous populations, *J. Math.Biol.* 28(1990) 365**.**

Endemic Equilibrium Equations

$$I_S{}^* = \frac{\frac{\lambda ABCN}{\beta} - \mu N(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)}{\frac{\omega}{(\lambda+\omega)}\left\{\gamma_S(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A) + \frac{\gamma_{AS}C}{D}(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)\right\} - ABC\big((V) + (NV)\big)}$$

$$I_{AS}{}^* = \frac{\left(\frac{C(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)}{D(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)}\right)\left\{\frac{\lambda ABCN}{\beta} - \mu N(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)\right\}}{\frac{\omega}{(\lambda+\omega)}\left\{\gamma_S(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A) + \frac{\gamma_{AS}C}{D}(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)\right\} - ABC\big((V) + (NV)\big)}$$

$$R^* = \frac{\left(\left(\frac{\gamma_S}{(\lambda+\omega)}\right) + \left(\frac{\gamma_{AS}}{(\lambda+\omega)}\right)\frac{C(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)}{D(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)}\right)\left\{\frac{\lambda ABCN}{\beta} - \mu N(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A)\right\}}{\frac{\omega}{(\lambda+\omega)}\left\{\gamma_S(\sigma_{SV}(V)B + \sigma_{SNV}(NV)A) + \frac{\gamma_{AS}C}{D}(\sigma_{AV}(V)B + \sigma_{ANV}(NV)A)\right\} - ABC\big((V) + (NV)\big)}$$

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Visualization of SEIRS model: Attractor Plots

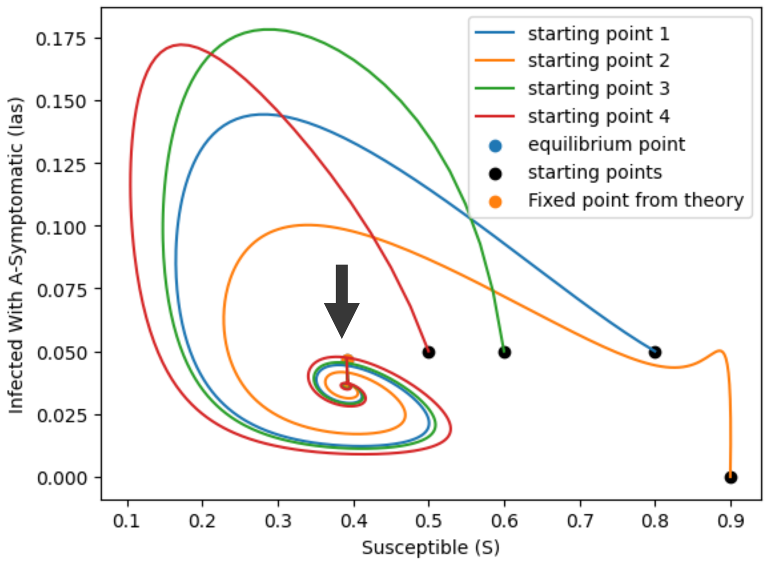
- Alternate visualization where states are compared as they evolve over time

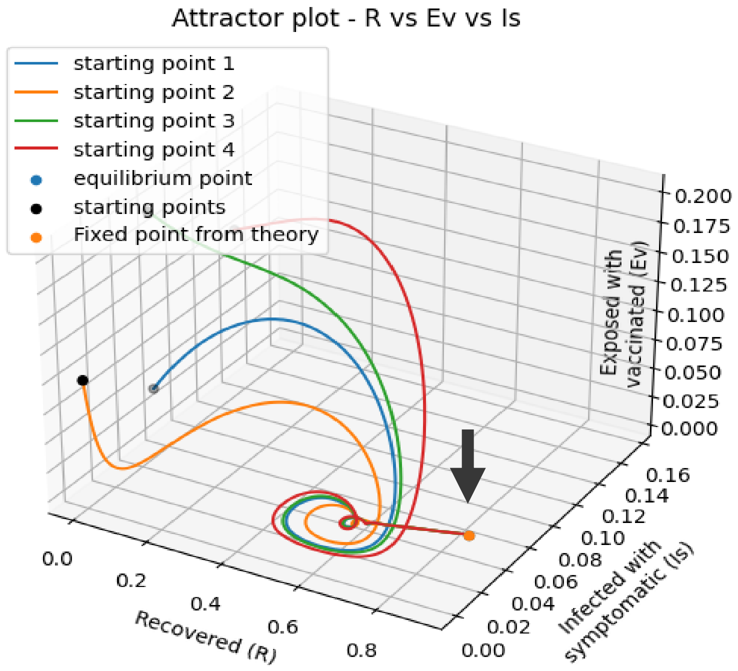
- Attractor plots can capture the long-term behavior of a system, showing the states or trajectories that a system tends to converge to

- Attractor plots can assist us in identifying the equilibrium points of the system

Mathematical Modelling of HIV/AIDS Dynamics with treatment and vertical transmission. Applied Mathematics 2012. 2(3): 77-89 doi: 10.5923/j.am.20120203.06

Example: Attractor Plots





Conclusion

- Developed a compartmental model for COVID-19 by considering the re-infection and vaccination.
- Analysed the equilibrium points with disease-free and endemic conditions.
- Visualization of SEIRS model using attractor plots.

Future Work

- Will work on the stability analysis for the modified SEIRS model.

Thank you!