Estimation of Extreme Values in Distributed Networks

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***Abstract*— This project studies the problem of distributed estimation of the extreme values such as the maximum and the minimum of the initial measurements at nodes in a distributed network without a fusion center. In the proposed algorithm, a nonlinear average consensus together with pre-processing and post-processing steps based on soft maximum functions is used. We show that all nodes will reach consensus on the estimate and the bias can be made small if a design parameter is set to be large. Simulations corroborating the results are provided.**

I. Project Description

Distributed networks with no fusion center have attracted many researchers due to broad advantages such as scalability [1-3]. In this work, we consider the problem of estimating the extreme values such as the maximum and minimum of the initial measurements in a distributed manner. This functionality in useful in various applications such as temperature control and network maintance [4].

The proposed algorithm is based on nonlinear average consnesus and soft maximum functuions to pre-process and post-process the states at the network nodes. The algorithm can be expressed in three phases: i) Assume x_i is the initial measurement at node i. In phase I, we pre-process the initial measurements at the nodes. The initial state at node i, denoted as $y_i(0) = \exp[\beta x_i]$, where $\beta > 0$ is a design parameter. ii) In phase II, nonlinear dustributed average consensus is applied in the network. The updating rule at node i at time $(t+1)$ can be expressed as,

$$y_i(t+1) = y_i(t) - \alpha(t)\left[d_i h(y_i(t)) - \sum_{j\in Ni} h(y_i(t)) + n_i(t)\right],$$

where $\alpha(t)$ is the step size, $h(.)$ is a nonlinear function to bound the transmission power, and $n_i(t)$ is the communication noise. By appyling the iterative algorithm, the states of the nodes are converging to the average of the initial states. iii) Assume consensus is reached at time t^*, the estimate of the maximum of the initial measurements can be obtained by processing the convergence result, i.e.,

$$\hat{x}_{maxi}(t^*) = \frac{1}{\beta}\,(logN + \log y_i(t^*)).$$

From the properties of soft maximum functions, we can see that the estiamte is always larger than the true maximum and the bias will be small if β is chosen to be large. The nonlinear bounded function is used to bound the transmission power because the power will be large when β is large when amplify and forward is used. Note that for $\beta < 0$,the minimum initial measurement can be estimated. Figure 1 shows results of max estimate with design parameter β=5.

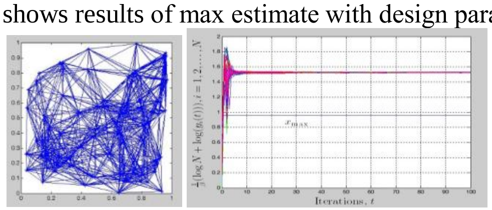

Fig. 1. Network structure and max estimate with β=5.

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From the figure we can see that the max estimate can be obtained in a few iterations but it is not accurate.Simulation results for the max estimate versus time with β = 6 and 7 are shown in Figure 2. From the figure we can see that a more accurate max estimate can be obtained with larger β, but the convergence time is also longer. Additional work on wireless sensor networks presented in [5-14].

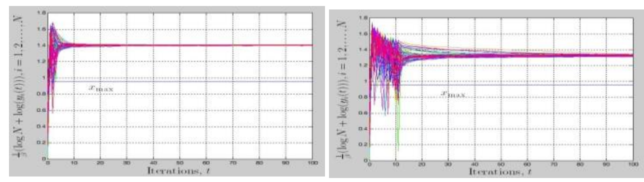

Fig. 2. Max estimate with β=6 (left) and β=7 (right).

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