Cramer-Rao lower bound Analysis in Sequential Sensor Localization

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***Abstract*— Localization accuracy is crucial in wireless sensor networks (WSNs). Localization in WSNs involved nodes usually at unknown locations and anchors at known locations. In this paper, a localization problem in a WSN with *M* anchors and *N* nodes is considered. In order to perform localization with fixed power requirements and limited communication range within the WSN, a sequential localization scheme is used, where anchors are used to find nodes, and nodes whose locations have been estimated, can be used as anchors for subsequent localization steps. In this problem, we specifically consider time of arrival (TOA) as the modality for localization. The Cramer-Rao lower bound (CRLB) on error in localization estimation is derived for sequential localization.**

***Index Terms*—sensor networks, localization, Cramer-Rao lower bound.**

I. Project Description

Consider a WSN with M anchors and N nodes, only one node is localized at each time. Once the node is localized, it is used as an anchor to localize the next node. Figure 1 shows the system model for sequential localization in 1-D. In the WSN, there is a total of M anchors and N nodes. To localize node 1, the node communicates with all M anchors. After node 1 is localized, it becomes an anchor. Therefore, when locating the node 2, besides M anchors, the distance between node 1 and node 2 is also measured. In the last step, all anchors and $N-1$ nodes communicates with the N^{th} node to estimate the N^{th} node location. In this case, TOA measurements are assumed to be Gaussian distributed [1,2]:

$$\hat{t} \sim \mathcal{N}\left(\frac{d_{i,j}}{c}, \sigma^2\right), \tag{1}$$

where d_{ij} is the distance between node i and anchor j, c is the speed of propagation and σ^2 is the variance of the noise. Since each node communicates with all anchors and all previously localized nodes, the Fisher information is the summation of two parts, one from the anchors, and another from the previously localized nodes. Define l_{ij} to be the log-likelihood function of the Gaussian distribution in (1). Define $\mathbf{F}$ as the Fisher information matrix with components Fz and Fa. Fa is the Fisher information from all anchors:

$$\mathbf{F}_a = \begin{cases} -\sum_{k=1}^{M} E\left[\frac{\partial^2}{\partial z_i^2} l_{ik}\right], & i = k \\ 0, & i \neq k \end{cases}, \tag{2}$$

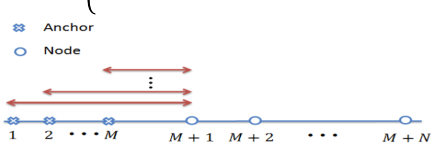

Figure 1. Sequential localization in one dimension. M anchors are used to localize node M+1. Once that is completed, the M anchors and node M+1 are used to localize node M+2, and so on.

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and $\mathbf{F}_z$ is the Fisher information from the first node 1 to the $(i-1)^{\text{th}}$ node, which is given as:

$$\mathbf{F}_z = \begin{cases} -\sum_{k=1}^{i-1} E\left[\frac{\partial^2}{\partial z_i^2} l_{ik}\right], & i = k \\ -E\left[\frac{\partial^2}{\partial z_i z_k} l_{ik}\right], & i \neq k \end{cases}. \tag{3}$$

From (3), we can see that when $i=k$, since the i^{th} node receives Fisher information from all anchors and all previous localized node, the summation contains $i-1$ terms. Therefore, by calculating the log-likelihood function l_{ij} and substituting into (2) and (3), we form the total Fisher information matrix as

$$\mathbf{F} = \mathbf{F}_a + \mathbf{F}_z = \frac{1}{c^2\sigma^2}\begin{bmatrix} M & -1 & \cdots & -1 \\ -1 & M+1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & \cdots & \cdots & M+N-1 \end{bmatrix}. \tag{4}$$

From (4) we can see that the diagonal element increases one at a time, to yield:

$$\mathrm{CRLB}_{1\mathrm{D}}^{\text{sequential}} \geq \frac{M}{(M+N)(M-1)}, \tag{5}$$

whereas the centralized method has a CRLB of:

$$\mathrm{CRLB}_{1\mathrm{D}}^{\text{centralized}} = c^2\sigma^2\frac{M+1}{M(M+N)}, \tag{6}$$

to give the ratio:

$$\frac{\mathrm{CRLB}_{1\mathrm{D}}^{\text{sequential}}}{\mathrm{CRLB}_{1\mathrm{D}}^{\text{centralized}}} \approx \frac{M^2}{M^2-1} \geq 1, \tag{7}$$

indicating that the centralized method always outperforms the sequential localization approach. Additional follow up work was published in [3-8].

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