Array Signal Recovery based on Matrix Completion

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Abstract — **In array processing, if some sensors are turned off or are unable to work properly, we will likely loose signal information and the beam pattern will be distorted. In this paper, a sparse sampling method is proposed that relies on on Matrix Completion (MC) theory. This method will allow us to use the inner relationship between array elements to reconstruct our missing signal data.**

***Index Terms*—Array processing, sensor failures, Matrix Completion theory, low rank, nuclear norm.**

I. Project Description

SUPPOSE we have a uniform linear array with n sensors. The distance between two elements is d.

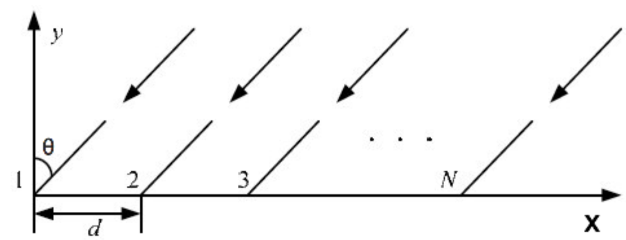

Fig. 1. A typical uniform linear array.

If there are m input signals, our received signal f at the i^th^ element [1-4] will be

$$x_i(t) = \sum_{h=1}^{m} s_h(t) \cdot e^{j\frac{2\pi}{\lambda_h}(i-1)d \sin \theta_h} + n,$$

where $s_h(t)$ are the original transmission signal with wavelength λ_h and direction of arrivals θ_h. Meanwhile, n is an additive white Gaussian noise (AWGN). If a few of our sensors are shut down, we may loose some signal information and our beam pattern may get distorted. Therefore, our target is to use the rest sensor signals to estimate the missing signals. So far, we commonly apply interpolation method or deep learning method to solve this problem [1][2]. Although those methods can estimate missing signals due to sensor failures, they are usually dependent on priori knowledge. Now, we introduce a signal recovery method based on Matrix Completion theory, which will help us to estimate the missing signals without any training data.

Matrix Completion (MC) [5-8] is the task to fill in the missing elements of a partially obtained matrix. Suppose our original integrated matrix is low rank. In certain occasions we may loose some elements in the matrix which will affect its rank. MC theory often seeks to find a way to refill the matrix and obtain the lowest rank. However, MC theory requires the matrix to follow strong incoherence property. That means when we loose an entire or entire column, the matrix cannot be effectively recovered. Therefore if some sensors in our array are turned off, several rows in our receiving signal matrix will become zero. Therefore, the traditional MC theory will not be feasible for our array signal matrix recovery.

To avoid loosing the entire row of our signal matrix, we will choose an appropriate time step t_0 and our received signal vector will therefore be

$$x(t_0) = [x_1(t_0), \cdots, x_i(t_0), \cdots, x_n(t_0)].$$

Now we can reshape this vector into a matrix [5]:

$$\tilde{x}(t_0) = \begin{bmatrix} x_1(t_0) & \cdots & x_p(t_0) \\ \vdots & \ddots & \vdots \\ x_{p(q-1)+1}(t_0) & \cdots & x_n(t_0) \end{bmatrix}_{p \times q},$$

where $p \times q = n$. For a fixed ULA, the elements in the steering vector

$$a(\theta) = [1, \cdots, e^{j\frac{2\pi}{\lambda}(i-1)d \sin \theta}, \cdots, e^{j\frac{2\pi}{\lambda}(n-1)d \sin \theta}]$$

will be a geometric sequence. So the matrix $\tilde{x}(t_0)$ will approximately be low rank and the rank will equal to the source number.

Now to reconstruct our missing signals, we can apply MC theory to recover our signal matrix at certain time steps. Since minimizing the rank of the matrix is generally NP-hard [9], we can try to transform this problem to minimize the nuclear norm of the matrix. After a semidefinite programming (SDP) process, this problem will be transformed into a convex optimization problem and can be solved by existing interior point methods.

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