INTRODUCTION AND MOTIVATION

- Wireless sensor networks (WSNs) with no fusion center
- Estimate the system size (total number of sensors) of WSNs
- Applications: network maintenance, detect nodes join or leave the network

PROBLEM STATEMENT

- Count number of nodes using consensus
- Random initial values, $x_i$, generated at nodes
- Different consensus algorithms:
  - Max consensus
    \[ x_i(t + 1) = \max \left\{ x_i(t), \max_{j \in N} x_j(t) \right\} \]
  - Average consensus
    \[ x_i(t + 1) = W_{ii} x_i(t) + \sum_{j \neq i} W_{ij} x_j(t) \]
- System size inferred from the consensus results
- Problem: performance is affected by:
  - Types of consensus algorithm
  - Initial values, $x_i$
- Performance analysis: Fisher information (FI) and Cramer-Rao bounds (CRBs)

RESULTS: FI AND CRBS UNDER DIFFERENT CASES

Max Consensus in the Absence of Noise

Theorem

Assume the initial values at nodes $x_i$ are i.i.d.
with $F(x)$ and $CDF F(x)$, and $f(x)$ is differentiable. When max consensus is used, the Fisher information for estimate of system size $N$ is,

\[ I_{max} = \frac{1}{N^2}. \]

The CRB is the inverse of the Fisher information, and a lower bound on the estimation variance can be expressed as

\[ \text{Var} \left[ \hat{N} \right] \geq N^2. \]

The distribution of the initial values at nodes does not affect the Fisher information and CRB.

Average Consensus in the Absence of Noise

Theorem

Assume the initial values at nodes $x_i$ are i.i.d. with mean $\mu$ and variance $\sigma^2$. Also assume that $N$ is large. When average consensus is used, the Fisher information for estimate of system size $N$ is

\[ I_{ave} = \frac{1}{2N}. \]

The CRB is the inverse of the Fisher information, and an lower bound on the estimation variance can be expressed as

\[ \text{Var} \left[ \hat{N} \right] \geq 2N^2. \]

Max Consensus with Noise

Theorem

Assume the initial values at nodes $x_i$ have exponential tail and its tail PDF $\lambda e^{-\lambda x}$. The distribution of the max of the initial values can be approximated using Gumbel distribution. Assume that the final error at nodes is Gaussian distributed $e \sim N(\mu, \sigma^2)$. The Fisher information for estimate of system size $N$ is bounded by

\[ I_{max} \leq \left( \frac{1}{N^2} \right) \left( \frac{\lambda^2}{\sigma^2 + \lambda^2} \right). \]

The CRB is the inverse of the Fisher information, and a lower bound on the estimation variance can be expressed as

\[ \text{Var} \left[ \hat{N} \right] \geq N^2 \left( \sigma^2 + \lambda^2 \right). \]

Average Consensus with Noise

Theorem

Assume the initial values at nodes $x_i$ are i.i.d. with mean $\mu$ and variance $\sigma^2$. The final error at nodes is Gaussian distributed $e \sim N(0, \sigma^2)$. Also assume that $N$ is large. When average consensus is used, the Fisher information for estimate of system size $N$ is

\[ I_{ave} = \frac{1}{2N} \left( \frac{\sigma^4}{\sigma^4 + \sigma^2} \right). \]

When $N$ is large, $I_{ave} \approx \frac{1}{2N \sigma^2} \approx \frac{1}{2N} (\text{SNR})$, where SNR is defined as $\frac{\sigma^2}{\sigma^2}$. The CRB is the inverse of the Fisher information, and an lower bound on the estimation variance can be expressed as

\[ \text{Var} \left[ \hat{N} \right] \geq 2N^2 \left( \frac{\sigma^4}{\sigma^4 + \sigma^2} \right). \]

CONCLUSIONS

- For max consensus without noise, $x_i$ does not affect Fisher information and Cramer-Rao Bound
- Max consensus has lower CRB (noiseless case)
- How noise affect FI and CRB is presented

REFERENCES


ACKNOWLEDGMENT

This work is funded in part NSF award ECCS – 13079282 and the SenSIP Center, School of ECEE, Arizona State University

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