Sparse Models vs. Representations

- Better encoding or "Representation" schemes perform better for a given best sparse model called the "Dictionary".
- Recent results show that a simple k-means based dictionary or just using random examples from the training data perform well [1].

Weak Sparse Models:
- Weaker models from small subsets of training data.
- Can we combine multiple weak sparse models to improve performance?

Weak Representations:
- 1-sparse representations / Correlate-and-Maximize.
- Can we combine multiple weak representations from weak sparse models to improve performance?

Ensemble Approach:
- For the data x, compute L sparse approximations, each using a different dictionary \( D_i \):

\[
\{D_1a_1, D_2a_2, \ldots, D_La_L\}
\]
- The ensemble approximation of \( x \) is the linear combination

\[
\sum_{i=1}^{L} D_i a_i
\]
- Parameters of the ensemble

Advantages of Ensemble Approach[2]:
- Statistical: Better approximation of test data.
- Computational: Avoid local optimal in large datasets.
- Representational: Overcomes hypothesis space limitations of single model.

Application: Image Restoration

- Degradation Aware Boosting:
  - Random-projection based compressive sensing. Measurement matrices have i.i.d. Gaussian entries.
  - Boosted dictionaries optimized to degradation. Knowledge of only the form of degradation is sufficient.

Weak Dictionaries Used:
- Boosted Example Selection (BoostEx)
  - In each round, \( D_i \) is a normalized set of K examples chosen according to \( p(x_i) \)
- Boosted K-Means (BoostKM)

\[
\min_{(\mu_k)_{k=1}^K} \sum_{k=1}^K \sum_{i \in M_k} p(x_i) || x_i - \mu_k ||^2
\]
- Dictionary elements are just the normalized initial cluster centers.

Weak Representations and Boosting[3]:
- Greedy forward selection model
  \( x_i = (1 - \alpha_l) x_{i-1} + \alpha_l D_i a_i \)
- Final approximation is an affine combination of the individual approximations.
- The weights \( \{\alpha_l\} \) have a one-to-one mapping to \( \{\beta_l\} \).
- \( D_{L+1} \) is chosen to well-represent poorly approximated by \( D_L \).

Compressed Recovery Of Standard Images: PSNR (Db)

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