Estimation of Extreme Values in Distributed Networks

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Abstract — This project studies the problem of distributed estimation of the extreme values such as the maximum and the minimum of the initial measurements at nodes in a distributed network without a fusion center. In the proposed algorithm, a nonlinear average consensus together with pre-processing and post-processing steps based on soft maximum functions is used. We show that all nodes will reach consensus on the estimate and the bias can be made small if a design parameter is set to be large. Simulations corroborating the results are provided.

I. PROJECT DESCRIPTION

Distributed networks with no fusion center have attracted many researchers due to broad advantages such as scalability [1-3]. In this work, we consider the problem of estimating the extreme values such as the maximum and minimum of the initial measurements in a distributed manner. This functionality in useful in various applications such as temperature control and network maintenance [4].

The proposed algorithm is based on nonlinear average consensus and soft maximum functions to pre-process and post-process the states at the network nodes. The algorithm can be expressed in three phases: i) Assume $x_i$ is the initial measurement at node $i$. In phase I, we pre-process the initial measurements at the nodes. The initial state at node $i$, denoted as $y_i(0) = \exp[\beta x_i]$, where $\beta > 0$ is a design parameter. ii) In phase II, nonlinear distributed average consensus is applied in the network. The updating rule at node $i$ at time $(t + 1)$ can be expressed as,

$$y_i(t + 1) = y_i(t) - \alpha(t) \left[ d_i h(y_i(t)) - \sum_{j \in N_i} h(y_j(t)) + n_i(t) \right],$$

where $\alpha(t)$ is the step size, $h(\cdot)$ is a nonlinear function to bound the transmission power, and $n_i(t)$ is the communication noise. By applying the iterative algorithm, the states of the nodes are converging to the average of the initial states. iii) Assume consensus is reached at time $t^*$, the estimate of the maximum of the initial measurements can be obtained by processing the convergence result, i.e.,

$$\hat{\xi}_{\text{max}}(t^*) = \frac{1}{\beta} \left( \log N + \log y_{\text{max}}(t^*) \right).$$

From the properties of soft maximum functions, we can see that the estimate is always larger than the true maximum and the bias will be small if $\beta$ is chosen to be large. The nonlinear bounded function is used to bound the transmission power because the power will be large when $\beta$ is large when amplify and forward is used. Note that for $\beta < 0$ ,the minimum initial measurement can be estimated. Figure 1 shows results of max estimate with design parameter $\beta=5$.

From the figure we can see that the max estimate can be obtained in a few iterations but it is not accurate. Simulation results for the max estimate versus time with $\beta = 6$ and 7 are shown in Figure 2. From the figure we can see that a more accurate max estimate can be obtained with larger $\beta$, but the convergence time is also longer. Additional work on wireless sensor networks presented in [5-14].

Fig. 1. Network structure and max estimate with $\beta=5$.

Fig. 2. Max estimate with $\beta=6$ (left) and $\beta=7$ (right).

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