GLOBAL OPTIMIZATION OF GRAPH FILTERS WITH MUTIPLE SHIFT MATRICES

Jie Fan, Cihan Tepedelenlioglu, Andreas Spanias

SenSIP Center, School of ECEE, Arizona State University

MOTIVATION

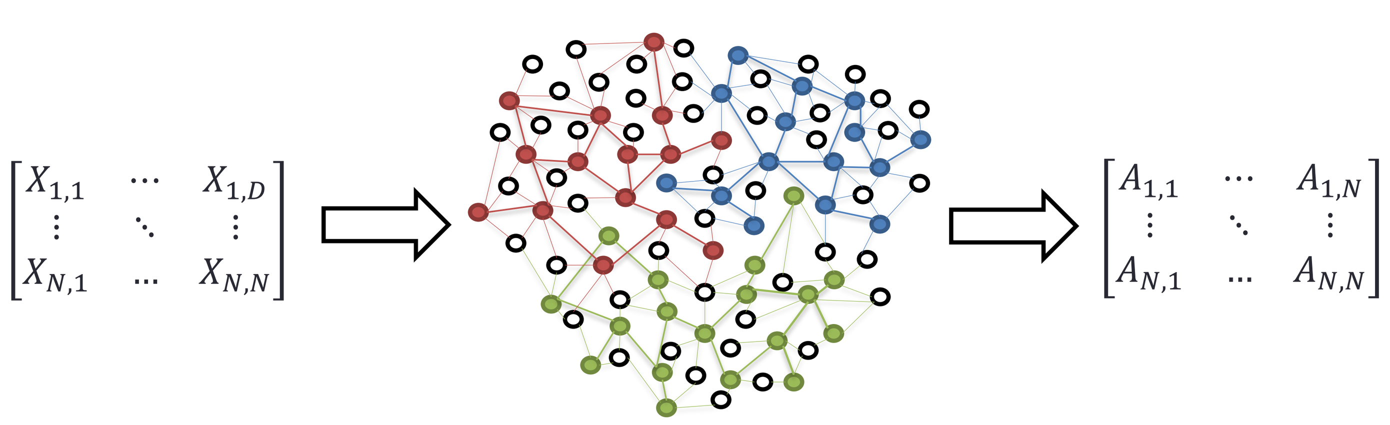
- Graphs can capture complex relational characteristics.
- Graph signal processing has advantage in dealing with datasets with irregular and complex structures.
- Adopting multiple shift matrices provides more flexibility in graph filter design.

POTENTIAL APPLICATIONS

- A classifier for data labeling.
- An error detector for network analysis.
- A pre-process of neural networks for reducing computation and mitigating overfitting risk.

PROBLEM STATEMENT

- A partially labeled dataset with graph encoded inner interaction.
  Graph vertices: data points.
  Graph edges: similarities among the vertices.
- The feature qualities of vertices are uneven.
- Graph shift matrices are generated from the dataset.
- Graph parameters are decided through branch and bound optimization method.
- A graph filter is designed as the classifier.



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GRAPH FILTERING PROCEDURE

- **Graph Filtering:**

$$\mathbf{S}^{\mathrm{cla}} = \mathrm{Q}(\mathbf{S}^{\mathrm{fil}}) = \mathbf{HS}$$

- **Conventional Graph Filter Design Method:**

$$A_{i,j} = \frac{\exp\left(-\frac{\rho(\mathbf{x}_i, \mathbf{x}_j)}{\sigma}\right)}{\sum_{i=1}^{N} \exp\left(-\frac{\rho(\mathbf{x}_i, \mathbf{x}_j)}{\sigma}\right)}$$

$$\mathbf{H} = h_0\mathbf{I} + h_1\mathbf{A} + h_2\mathbf{A}^2 + \cdots + h_L\mathbf{A}^L$$

- **Proposed Graph Filter Design Method:**

$$A(d)_{i,j} = \frac{\exp\left(-\frac{(\mathrm{x}_{i,d} - \mathrm{x}_{j,d})^2}{\sigma}\right)}{\sum_{i=1}^{N} \exp\left(-\frac{(\mathrm{x}_{i,d} - \mathrm{x}_{j,d})^2}{\sigma}\right)}$$

$$\mathbf{H} = \sum_{d=1}^{D} \sum_{l=1}^{L} w_d h_l \boldsymbol{A}(d)^l$$

$$\text{subject to } \mathbf{h} \in \Theta_{\mathrm{h}},\ \mathbf{w} \in \Theta_{\mathrm{w}}$$

- **Convex Relaxation**

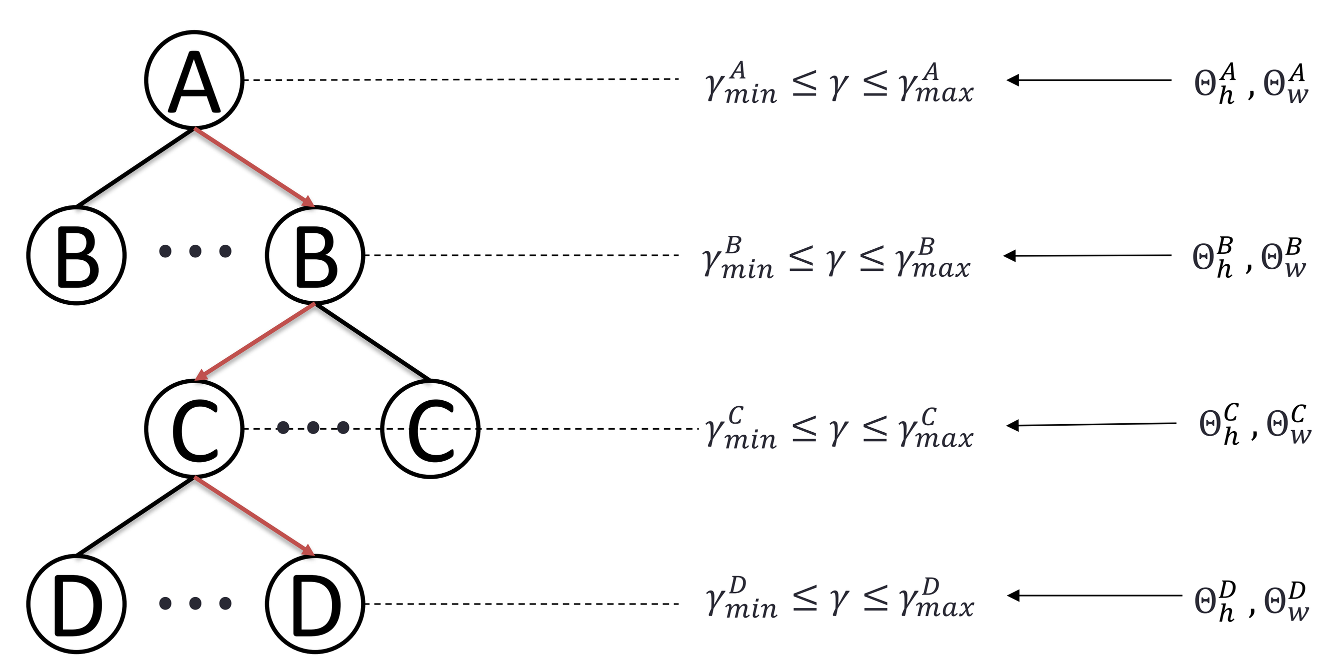
$$L = \arg\min_{\mathbf{h},\mathbf{w}} \left\| \sum_{d=1}^{D} \sum_{l=1}^{L} \gamma_{d,l} (\mathbf{RA}(d)^l \mathbf{S}) - \mathbf{S} \right\|_F$$

$$\gamma_{d,l} = w_d h_l$$

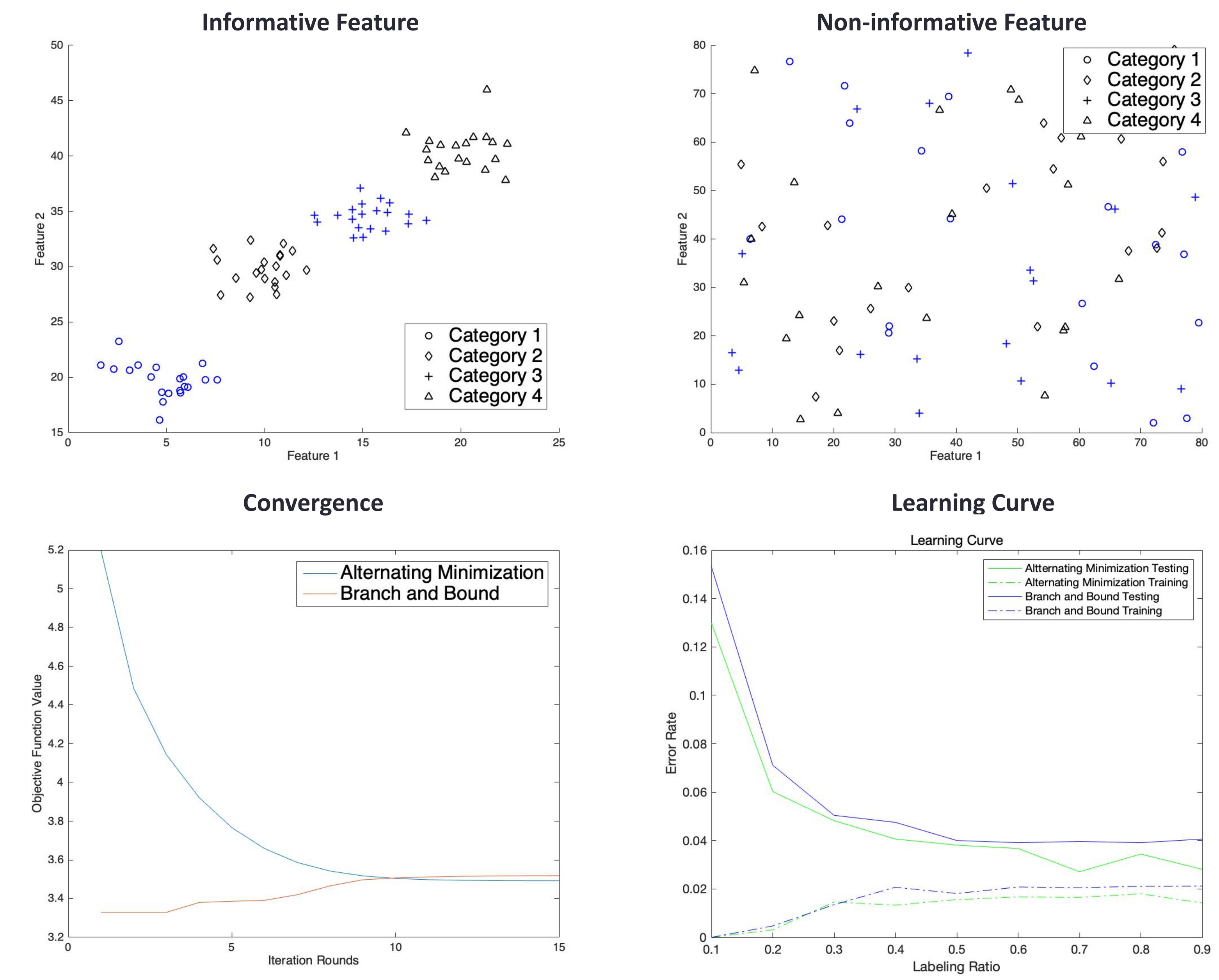
$$\gamma_{d,l} \ge \max(w_d^{min} h_l + h_l^{min} w_d - w_d^{min} h_l^{min}, w_d^{max} h_l + h_l^{max} w_d - w_d^{max} h_l^{max})$$

$$\gamma_{d,l} \le \min(w_d^{max} h_l + h_l^{min} w_d - w_d^{max} h_l^{min}, w_d^{min} h_l + h_l^{max} w_d - w_d^{min} h_l^{max})$$

BRANCH AND BOUND



SIMULATION DATA WITH UNEVEN FEATURES



CONCLUSION

- A well designed graph filter can work as a semi-supervised classifier.
- The proposed filter designing method provides lower error rate than the conventional one when feature qualities are uneven.
- The branch and bound method can technically provide the global optima for our nonconvex problem and then a benchmark can be provided.

REFERENCES

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