A Brief Survey of Time- and Frequency-Domain Adaptive Filters

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Abstract—This tutorial session paper provides a brief survey of adaptive signal processing methods and applications. We give the update equations of FIR and IIR adaptive filters. We provide brief descriptions of LMS, RLS, and block time and frequency domain algorithms. The tutorial begins with an introduction to adaptive methods and continues with a discussion on least squares and gradient techniques. The session covers various adaptive filter structures and provides information and references on several applications including system identification, linear prediction, noise cancellation, and adaptive arrays.

Keywords—Adaptive Filters, Linear Prediction, LMS, RLS, System Identification, Noise and Echo Cancellation, Tutorial in Adaptive Signal Processing.

BRIEF SURVEY

This section provides a brief survey of adaptive algorithms for filtering applications. Discrete-time adaptive signal processing (ASP) algorithms [1-5] and more specifically Least Mean Square (LMS) [1] and recursive least square adaptive filtering methods have found several applications including: noise [6-10] and echo cancellation [11-20,55,160], channel equalization [21-28,122], active noise control [9,60], adaptive arrays [29-33], biomedical signal analysis [34,35,61,116,209], adaptive prediction [36-41], and system identification [42-45,158,161]. We begin our tutorial with the adaptive linear combiner [1] shown in Fig. 1. The error is the difference between the desired signal and the filter output. Mean square error minimization gives the Wiener solution [1], i.e., \( b = R_{xx}^{-1} r_{dx} \), where \( b \) is the weight vector, \( R_{xx} \) is the autocorrelation matrix and \( r_{dx} \) is the crosscorrelation vector.

The sequential LMS equation for adapting \( b \) is given by

\[
b(n+1) = b(n) + 2\mu x(n) e(n)
\]

where \( \mu \) is the step size or convergence factor. The FIR coefficient vector converges in the mean to the Wiener solution if \( 0 < \mu < 1/\lambda_{\text{max}} \), where \( \lambda_{\text{max}} \) is the largest eigenvalue of the autocorrelation matrix. Under a sequence of assumptions, it can be shown that misadjustment is approximately: \( M \approx \mu \text{trace}(R_{xx}) \) [1]. The step size in many applications is typically normalized [45]. Optimal step sizes can also be derived for LMS algorithms [46, 181]. Block LMS (BLMS) adaptive algorithms [47] process data one block at a time and the gradient is computed from a block of error samples. The error vector and update equations for the BLMS are given by:

\[
e(i) = d(i) - x'(i)\hat{b}(i); \quad \hat{b}(i+1) = \hat{b}(i) + \frac{2\mu}{N} x'(i) e(i).
\]

Block algorithms with optimal step size were proposed in [48]. All the sequential algorithms with a single step size have slow convergence when \( R_{xx} \) has large eigenvalue spread. Recursive Least Squares (RLS) type algorithms [2,3,50,51] generally converge more rapidly than LMS filters but also have considerably higher complexity. An RLS algorithm can be formulated using the matrix inversion lemma. The RLS updates filter coefficients using the following equation:

\[
b(n+1) = b(n) + k(n+1) e(n+1)
\]

where the gain \( k(n+1) \) is given by

\[
k(n+1) = \frac{b(n)' R_{xx}^{-1}(n) x(n+1)}{1 + \beta b(n)' R_{xx}^{-1}(n) x(n+1)}
\]

Modified and fast RLS algorithms have been proposed in [11,52,53,55,176]. Fast RLS for nonlinear Volterra filters have been proposed in [56-59]. RLS can be viewed as a special case of a Kalman filter [2,195]. Lattice structures for adaptive FIR filters and for linear prediction systems have been proposed in [63-66,145,184-187] with demonstrated advantages in fixed-point implementations [66]. Frequency-domain adaptive algorithms [67] use the FFT for fast convolution, Fig. 2.

Fig. 1. The FIR Adaptive Filter.

Fig. 2. The Frequency-Domain Adaptive Filter.
The update equation in this case is based on the complex LMS algorithm [68]. Computation of the output of the filter and the gradient is based on circular convolutions [70]. Analysis of frequency domain LMS algorithms was presented in [70]. FFT-based adaptive FIR algorithms that use the overlap and save method for output and gradient computation are presented in [71,72] and with optimal step size in [43,44]. Unconstrained FFT-based adaptive FIR algorithms are given in [73]. Relations of adaptive multirate filters and their frequency domain counterparts are established in [74]. Adaptive frequency-domain algorithms for multichannel noise reduction have been presented in [169]. IIR frequency domain adaptive filters are derived in [75]. Relations between the LMS and the running DFT have been derived in [144]. IIR adaptive filters are more difficult to analyze and implement because of the inherent nonlinearity of the error w.r.t. feedback parameters [78-91,172,177,189]. Simple adaptive IIR filters use the equation error (Fig. 3) model [77]. Although the equation error sequence is linear w.r.t. the feedback parameters, the algorithm converges to a biased parameter vector.

Fig. 3. The Equation Error IIR Adaptive Filter.

The error, update, data vector, and coefficient vector of the equation error IIR adaptive filter are given by

\[ e(n) = d(n) + g^T(n)c(n), \quad c(n+1) = c(n) + 2\mu e(n)g(n). \]

\[ g(n) = [x(n) - x(n-L) \ldots - x(n-M)]^T, \]

\[ c(n) = [b_0(n), b_1(n) \ldots b_L(n), a_0(n), a_1(n) \ldots a_M(n)]^T. \]

Algorithms based on the output error model (Fig. 4) have also been proposed in several papers. One of the simplest ones is the Simple Hyperstable Adaptive Recursive Filter (SHARF) [84].

Fig. 4. SHARF Output Error IIR Adaptive Filtering.

Some more recent work in IIR adaptive filtering suggests using genetic algorithms [83]. All-pole models for filter synthesis can be obtained by making \( B(z) = 0 \) in the structure of Figure 3. It can be shown easily that by minimizing the MSE with \( B(z) = 0 \), in the structure of Figure 3, the system essentially becomes equivalent to linear prediction (LP) [37,38]. Adaptive Linear prediction algorithms have been proposed for spectral analysis [92,93] and other applications. Adaptive linear prediction has found applications in speech and audio coding [94-97] and more specifically ADPCM [98-100]. RLS “batch” type algorithms for linear prediction, and more specifically Levinson-Durbin type algorithms [37] are at the core of most LPC and CELP speech codecs [39,124]. Applications of LP to genomics [125], simulation environments for LP [126,127], applications to EMG [209] and extensions to pole-zero prediction [40] also appeared in the literature. Reference [37] is a comprehensive tutorial for LP.

Differential coding also uses a variant of LMS called the sign error LMS which also found applications in other quantized filter and control implementations. To that end, there are several variants of LMS algorithms [101,210]. Publication [101] and references therein describe several LMS variants including the normalized LMS, the leaky LMS, the LMS with dead-zones, the sign-sign LMS, and the median LMS. Other LMS type algorithms and fixed point effects have been reported in [130-134,161,168,208]. LMS algorithms with bounded error constraints have been proposed in [42,110,111]. LMS algorithms have been used in hard drive equalization [112,123] and hard drive control [113]. Diffusion and consensus based adaptive techniques have been proposed in [135-139,146-150,182]. In addition to diffusion techniques, adaptive methods using kernel mapping, have also been developed. [151]. Learning systems and adaptive gradient and RLS methods have also been used in neural networks [152-154]. The x-filtered LMS is used in several control applications and has been applied in active noise cancellation [7,9,80,114,156,157,162]. LMS algorithms for 2-dimensional processing are described in [102-106]. Filter bank and sub-band adaptive algorithms [107-109,117-121,140-143,155,159] have been developed and deployed in audio and image/video applications. Adaptive filters with sparse impulse responses (Fig. 5) have been useful in echo cancellation and other applications [115,128,129,202].

Fig. 5. Sparse adaptive signal processing; a) sparse impulse response, b) simple sparse adaptive filter.

Sub-band adaptive filters can be implemented using multirate structures (Fig. 6) which allow separate step-sizes and customized adaptation strategies for each sub-band.

ADAPTIVE SIGNAL PROCESSING APPLICATIONS

As mentioned before there are several applications of adaptive algorithms including adaptive noise and echo cancellation [205,207], channel equalization [21,188], adaptive line enhancement [178,179,180], active noise control [156], sensor arrays [170,171], estimation of head related transfer functions [166], hearing aids [163-165], sound processing and effects [167,173,174,175], machine learning [183], network applications [148] and neural nets [154]. Array processing
using LMS and RLS algorithms has been used in several areas including adaptive antennas, radar, and linear and circular microphone sensor arrays. The weights of the array [170] control the beamwidth, sidelobes and directivity.

An LMS algorithm with window constrains has been used to adapt the beam directivity while keeping low level sidelobes [192]. This is achieved by forcing constrains on the LMS array parameters (Fig. 7).

Fig. 6. Simple two sub-band adaptive algorithm.

Adaptive arrays have also been used for acoustic signal localization [175], MIMO antenna systems [204], and GPS [203]. Microphone arrays [171,196-201] have also been used in multichannel adaptive echo cancellers, active noise control systems (Fig. 8), and smart internet of things (IoT) devices. In particular, adaptive circular microphone arrays [200, 201] are embedded in modern IoT speakers.

Fig. 7. LMS beamforming with window constraints for beam steering with suppressed sidelobes [192].

Adaptive algorithms have also been used in subspace Wiener filters [193] and in orthogonal frequency-division multiplexing (OFDM) systems for channel estimation. In OFDM each channel is assumed constant for each symbol. The received signal is a combination of the transmitted signals. Additive noise is assumed and frequency-domain channel equalization is part of the system. The channel estimates can be obtained with the LMS or RLS algorithms [204]. DFTs and IDFTs are implemented efficiently with FFT algorithms [204].

Fig. 8. Experimental Platform for adaptive multichannel active noise control [194].

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Fig. 9. OFDM with LMS Channel Estimation [204].

A multichannel echo canceller with M speakers and N microphones is shown in Fig. 10 [206]. There are MN echo acoustic paths to be estimated. DFTs are used for decoupling and the transformations G are used to decompose MIMO modes into single input/single output (SISO) modes. Efficient methods for estimating the echo paths are presented in [206].

Fig. 10. MIMO Adaptive Echo Canceller [206].

CONCLUSION

This adaptive signal processing tutorial summary described briefly some of the basic methods and applications of adaptive filtering. The paper gave short overviews of algorithms and provided an extensive list of references.

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A weighted criterion.


