Max Consensus in the Presence of Additive Noise

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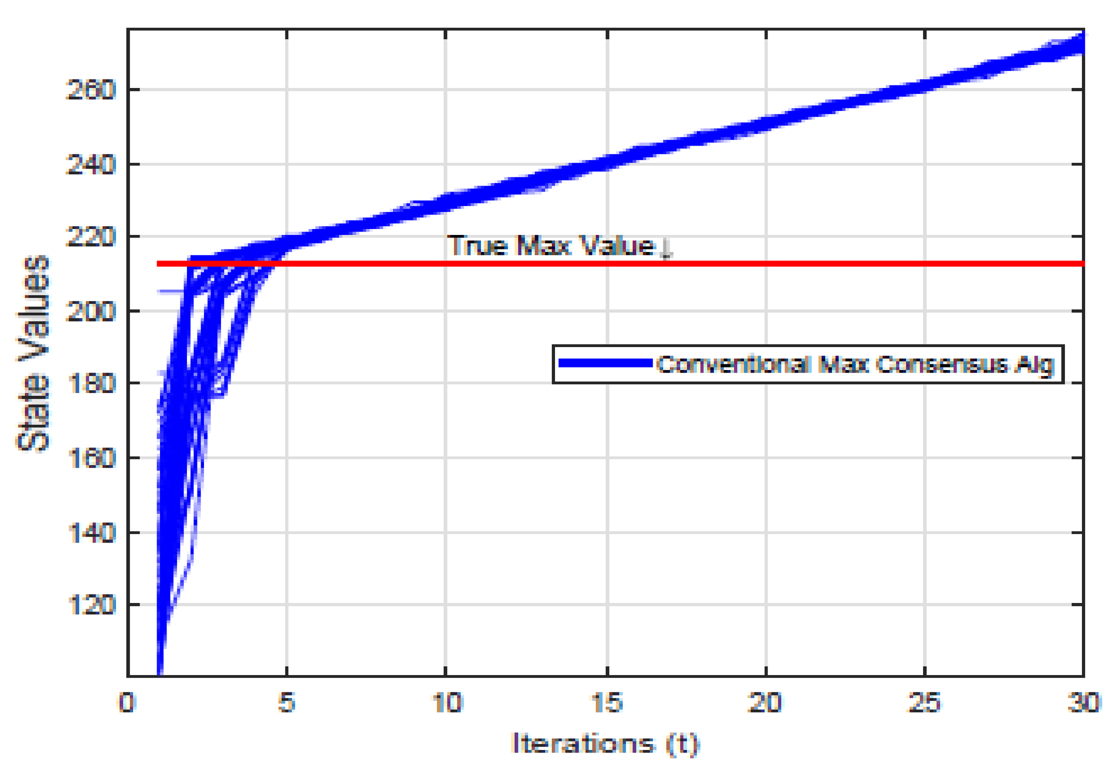
MOTIVATION

- **Fully distributed estimation of maximum of initial node state values in the presence of additive noise.**
- **Due to additive channel noise, the estimate of the maximum at each node has a positive drift at each iteration and this results in nodes diverging from the true max value.**
- **Derive bounds on growth rate using large deviation theory.**
- **Design a fast max based 2-run algorithm robust to additive noise, which compensates for the growth rate.**



POTENTIAL APPLICATIONS

- **Distributed computation of extremes is necessary in :**

resource management, network area estimation, node counting, node clustering, message routing and distributed detection and tracking.

CONVENTIONAL MAX CONSENSUS

- **Each node updates their state value by the maximum of the received measurements. In the presence of additive noise the update equation is given by,**

$$x_i(t+1) = \max\Big(x_i(t), \max_{j \in \mathcal{N}_i}(x_j(t) + v_{ij}(t))\Big)$$

UPPER BOUND ON GROWTH RATE

- **Upper bound for general additive noise distribution is given by ,**

$$\lambda \leq \inf\left\{x : \sup_{0 \leq \beta \leq 1}\Big[H(\beta) + \beta\log(\rho) - \beta I(x/\beta)\Big]\right\}$$

- **Upper bound for additive Gaussian noise :**

$$\lambda \leq \sup_{0 \leq \beta \leq 1} \sqrt{2\beta(H(\beta) + \beta\log(\rho))}$$

- **Alternate upper bound is obtained by solving :** $I(x) = \log(\rho + 1)$
- **Alternate upper bound for Gaussian noise :** $\lambda \leq \sqrt{2\log(\rho+1)}$

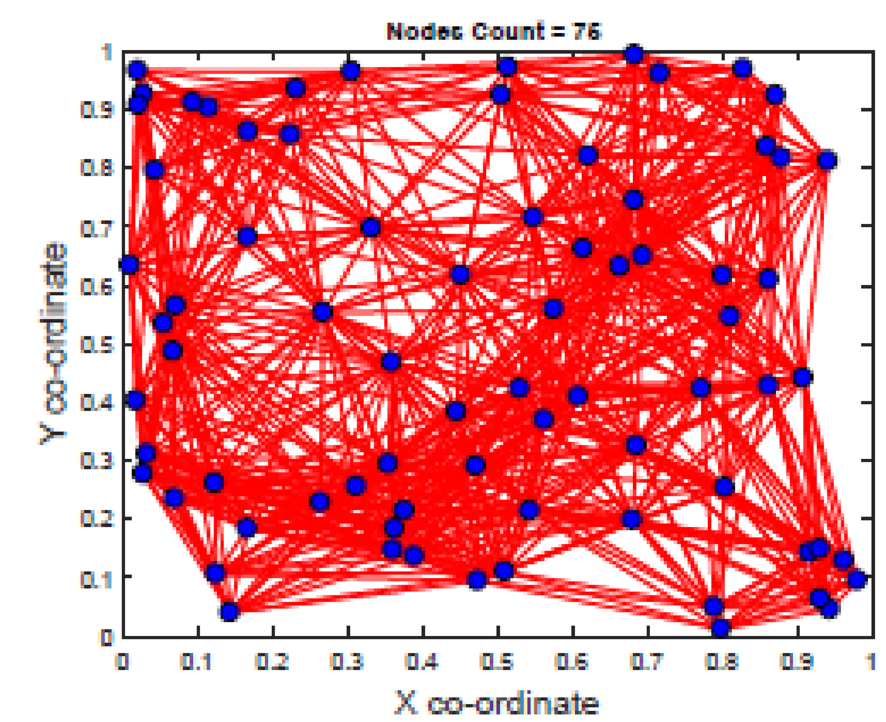
ROBUST MAX CONSENSUS ALGORITHM

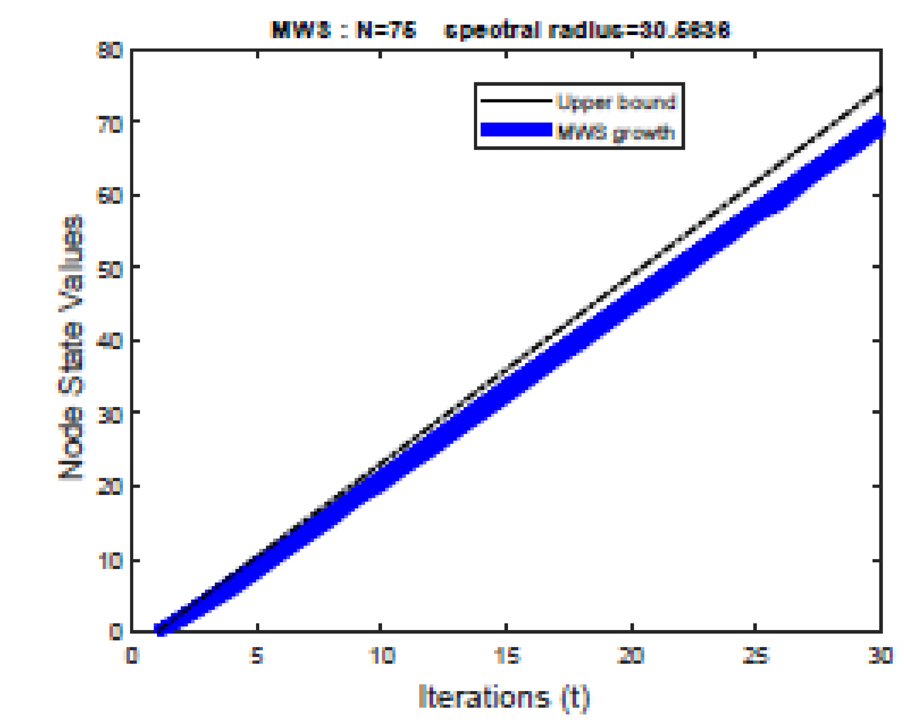
- **A two run algorithm is proposed to compensate the positive drift of the node state values (growth rate)**
- **First run : Initialize all nodes to zero and run the natural max consensus algorithm. After *t* iteration estimate the growth rate locally.**
- **Second run : With the knowledge of the local growth rate, run the max consensus algorithm on true state values. During each node update, subtract by the locally estimated growth rate.**

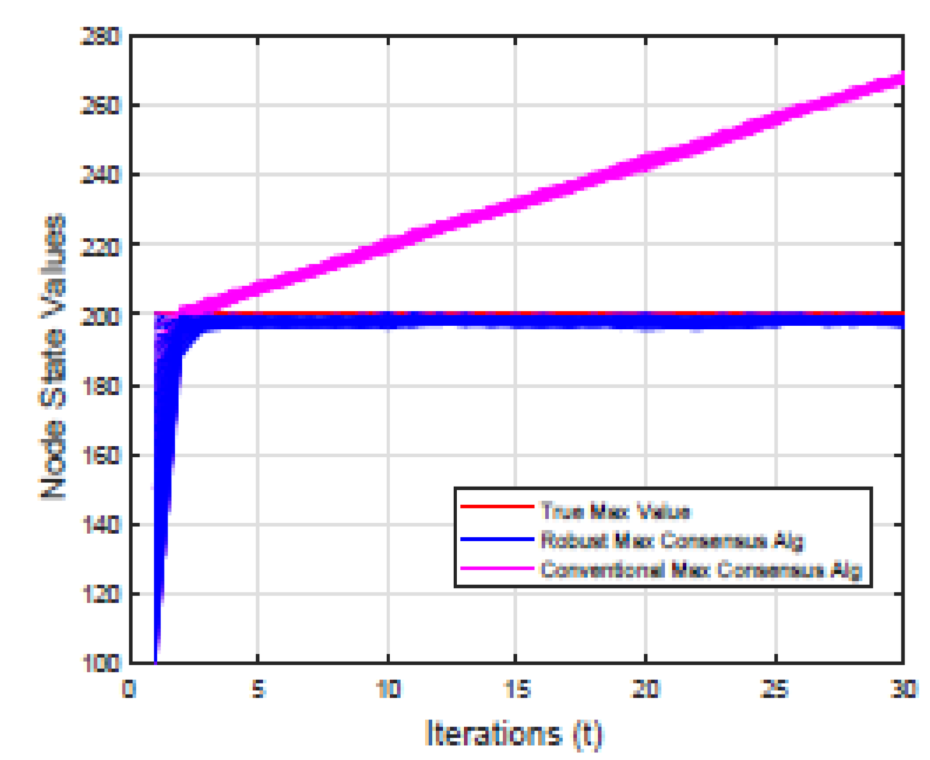
$$x_i(t+1) = \max\Big(x_i(t), \max_{j \in \mathcal{N}_i}(x_j(t) + v_{ij}(t))\Big) - \lambda_i.$$

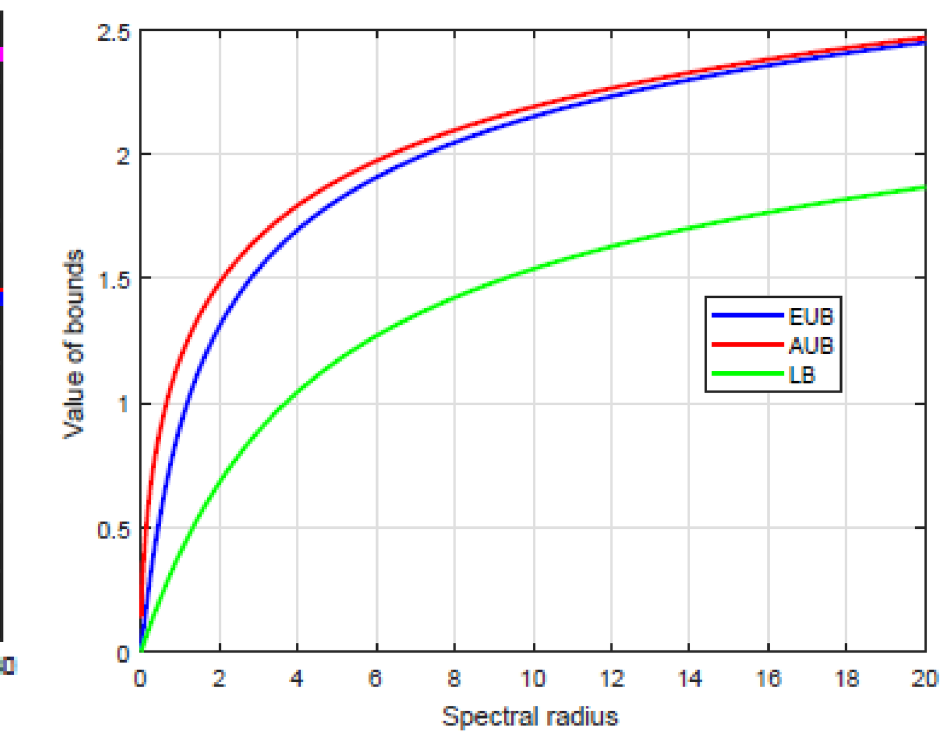
First run ::
  Input: iterations $= t$, # of nodes $= N$
  Initialization
   Initialize all nodes to zero, $x_i(0) = 0$
  repeat until : t **iterations**
   for $\{i = 1 : N\}$
   $x_i(t) = \max(x_i(t), \max_{j \in \mathcal{N}_i}(x_j(t-1) + v_{ij}(t-1)))$
   end : for
  end : repeat
  growth rate estimate : $\hat{\lambda}_i = \frac{x_i(t)}{t}$
Second run ::
  Input: # of nodes $= N$, Initial state : $x_i(0)$
  repeat until : convergence
  for $\{i = 1 : N\}$
  $x_i(t) = \max(x_i(t), \max_{j \in \mathcal{N}_i}(x_j(t-1) + v_{ij}(t-1))) - \hat{\lambda}_i$
   end : for
  end : repeat

SIMULATION RESULTS









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